Are Commodity and Stock Markets Independent of Each Other? *A Case Study in India*

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The history of the organized commodity derivatives market in India dates back to the 19th century, with the establishment of the first derivatives market in the form of Cotton Trade Association, where cotton futures contracts were traded in 1875, barely a decade after trading in commodity derivatives started in Chicago. Subsequently, derivatives trading started in oilseeds at Mumbai from 1900, in raw jute and jute goods at Kolkata from 1912, in wheat at Hapur from 1913, and in bullion at Mumbai from 1920. Later in 1939, in order to restrict speculative activity in the cotton market, options contracts in cotton were prohibited, and in 1943, forward trading in commodities including oilseeds, food-grains, spices, vegetable oils, sugar, and cloth was prohibited. After independence in 1947, the government enacted the Forward Contracts (Regulation) Act in 1952 to regulate forward contracts all over India in commodities, which are defined as any movable property other than security, currency, and actionable claims. The Act prohibited options trading in goods and cash settlement of forward trades, which severely affected the growth of the commodity derivatives market. Furthermore the Act allowed only those associations/exchanges that are recognized by the government to organize forward trading in approved commodities and also provided for three-tier regulation: the exchange that organizes forward trading in commodities to regulate trading on a day-to-day basis; the Forward Markets Commission to provide regulatory oversight under the powers delegated to it by the government; and the Ministry of Consumer Affairs, Food & Public Distribution, Government of India to be the ultimate regulatory authority. Consequent to repeated defaults on forward contracts during 1960s, forward trading was banned in many commodities. Later, in the 1970s and 1980s, the government relaxed forward trading rules for some commodities, but the market did not flourish.

During the liberalization era,[1] the government set up the K.N. Kabra Committee in 1993 to examine the role of commodity futures trading. The committee recommended allowing futures trading in 17 commodity groups, strengthening the Forward Markets Commission, and creating certain amendments to the Forward Contracts (Regulation) Act, in particular to allow options trading in goods and to register brokers with the Forward Markets Commission. The government accepted most of these recommendations, and trading in futures contracts was permitted in all recommended commodities. Further, the Ramamoorthy Committee appointed by the SEBI (Securities and Exchange Board of India) recommended fruitful cooperation between the commodity derivatives market and the stock market towards convergence of the two markets in terms of infrastructural facilities and the regulatory environment. Since 2002, the commodities futures market in India experienced an unprecedented boom with the setting up of multi-commodity exchanges that provide for electronic trading, a rapid increase in the number of commodities in which derivatives trading has been facilitated, and huge growth in trading volumes. On account of such developments, the commodity derivatives market in India has become as mature as the highly developed stock market in India. Against this background, the interactions in terms of price dynamics, if any, between the two markets in India merit qualitative and quantitative analysis.

COMMODITY DERIVATIVES AND STOCK MARKETS INTERACTIONS IN INDIA

The most important policy goal in commodity derivatives trading is safeguarding of the interests of producers (particularly farmers) as well as manufacturers, consumers, and other functionaries in the supply chain. Unlike the securities market, where the impact of the price volatility is on the willing participants in the market, the impact of a sharp rise or fall in the price of commodities is borne by the entire economy. If commodity derivatives markets function well, then some of the core policy goals of addressing volatility of agricultural prices may be addressed in a market-oriented fashion.

There is close resemblance between commodity derivatives and securities derivatives in so far as trade practices and mechanism are concerned. A commodity futures contract is tradable and fungible. Most of the commodity futures contracts are squared off and do not result in delivery. In this case, the users of commodity futures markets are using the contracts for purely financial purposes. Thus, almost all commodity futures contracts are akin to securities.

Though derivatives in commodities resemble securities and financial derivatives and provide many of the same economic functions, there are some major differences.

- There are actively traded spot markets for financial instruments, and prices are generally not discovered in the futures market. However, trading in commodity spot markets in India is restricted to consumption except for intermediary traders, as in any other country.
- As in many other developed economies, the spot market for securities in India is highly organized and effectively regulated by even agencies other than SEBI, such as the Department of Company Affairs, whereas the spot market for agricultural commodities is not so organized, though there are many laws to curb free markets in the agricultural sector.
- The settlement and delivery process in the two markets is different. While some international financial futures exchanges provide for cash or delivery based settlement, financial futures are fully cash-settled in India, whereas commodity futures are settled either in cash or in physical form. The moot point about cash settlement is that of well-respected and trusted settlement prices. If there is an underlying with a highly fractured spot market, where good data are not available, then it is difficult to construct a well-respected settlement price. In this case, a cash-settled contract will not be trusted and a physically settled contract will be preferred.
- The costs involved in dealing with physical goods (or warehouse receipts) are always higher than the costs of moving money. Further, the scale and mode of depositing/warehousing are structurally different.
- There are other supplementary legislations, such as the Depository Act, that make the functioning of stock markets smooth. In the case of commodity futures markets such supplemental institutions (like dematerialized warehouse receipts) are not available, except for a few exceptions in developed countries, which makes the delivery mechanism complex.
- Agricultural commodities have different shelf-life and demand–supply factors and price determinations. Precious metals also have different market conditions.
- Unlike the stock market, the factors affecting commodity prices are more complex and commodity-specific.
- As in any other nation, in India, indirect taxation cascades in commodities and income tax treatment are different from those applicable for equity investments. Loss due to speculation is not adjusted in corporate taxation in case of commodity futures but is only carried forward.

- The investor base and the number of registered brokers in the stock market are much larger when compared to the commodity derivatives market in India, as is the case in any market-driven economy.
- Indian financial institutions are not permitted to deal in commodity derivatives although they can invest in a restricted way in the stock market, in contrast to the freedom given to their counterparts in some advanced countries. Banks and financial institutions are considered stable institutions to provide market-making services, all over the world.
- Both commodity and financial derivatives are traded in the same exchanges worldwide, whereas in India, only financial derivatives are traded in stock exchanges and there are separate commodity derivatives exchanges.
- Both the spot and the derivatives segments of Indian stock exchanges are regulated by the securities market regulator SEBI. However, in line with the universal practice, the regulator of Indian commodity derivatives exchanges does not have jurisdiction over commodity spot markets even in nonagricultural commodities, such as bullion and other metals.

The possibilities of interactions are limited in so far as commodity futures trading requires highly specialized knowledge that is different from that required for securities trading. The firms that engage in commodity futures trading also differ from the firms that engage in securities trading.

IMPLICATIONS OF INTERACTIONS

The identification and quantification of causal relationships between the stock and the commodity derivatives markets, by analyzing the values over time of a market index and the commodity derivatives index, further the understanding of the markets' internal dynamics. Inter-linkage of the markets, if any, has a potential of providing growth impetus to commodity derivatives and opens new avenues of business opportunities to stock market participants, thereby deepening and broadening the markets. If a causal relationship from one market to the other is not detected, then informational efficiency exists in the second market. If causality is not found in both directions, then the two markets are independent of each other. The presence or absence of a causal relationship has a lot of implications including the following, for all the participants of the markets.

- At present, the government engages in many policy measures that interact with agricultural spot markets. These policies are unaffected by the question regarding the integration of commodity futures and stock markets. Whether the two markets are closely integrated has no impact upon the conduct of such policies as public procurement, support prices for commodities, and so on. To the extent that interactions between commodity derivatives and stock markets help strengthen price discovery on the commodity derivatives markets, this will facilitate the design of public policy. If shortages or gluts are expected to take place on a future date, this will be revealed in the futures price well ahead of time. This information will help policy-makers to respond proactively, if desired.
- If there is feedback in both directions, then investors may predict the behavior of one market using information on the other market. Since an impulse in a market is reflected quickly in the other market, policy intervention becomes more effective in the desired direction within reasonable time horizons.
- If the markets are not related, investors may reduce risk exposure by diversifying their portfolios across the markets.

METHODS TO STUDY THE INTERACTION

We have a set of simultaneously recorded variables—value of the stock index and the commodity derivatives index—over a period of time, and we want to measure to what extent these time series corresponding to such variables contribute to the generation of information and at what rate they exchange information. Various methods have been proposed for the simultaneous analysis of two series and generally cross-correlation and cross-spectrum are used for measuring relationships between such time series. However, these methods suffer from the drawbacks that they measure only linear relations (i.e., the nonlinear characteristics of the interactions between the markets represented by the two time series are not considered) and they lack directional information (i.e., they simply say how far the two market segments move together and do not identify the market segment where price discovery happens). Introducing time delay in the observations pertaining to one market segment may facilitate identifying an asymmetric relationship and hence direction of information flow, but nonlinear relationships will still remain unexplored.

Granger [1991] introduced an error-correction model that takes into account the nonstationary character of co-integrated variables and distinguishes between short-run deviations from equilibrium indicative of causal relationship and long-run deviations that account for efficiency and stability. This approach involves estimation of simultaneous linear equations in a pair of variables with time lags and has been used in a number of studies examining the causal structure of bivariate time series. Shanmugam and Prasad [2007] analyzed two years' data of crude oil prices in the Multi-Commodity Exchange of India (MCX) and the 30-stock Sensex index using regression analysis and found that an increase in crude oil prices led to a fall in the Sensex. They also reported that the equity prices of a few base metal companies and the associated metal futures prices in MCX are highly correlated.

A statistically rigorous approach to the detection of interdependence, including nonlinear dynamic relationships, between time series is provided by tools defined using the information theoretic concept of entropy, which is model independent (providing qualitative inferences across diverse model configurations). The basic concepts of entropy are given in the Appendix.

ENTROPIC MEASURES TO STUDY CAUSAL RELATIONSHIPS

Joe [1989] proposed relative entropy based measures of multivariate dependence for continuous and categorical variables, but these measures require the estimation of probability density or mass functions. Granger et al. [2004] proposed a transformed metric entropy measure of dependence for both continuous and discrete variables. Metric entropy is a measure of distance unlike relative entropy, which is a measure of only divergence, however the utility of metric entropy in studying statistical dependence based on causality is to be tested. The conditional entropies $H(Y/X) = H(X,Y) - H(X)$ and $H(X/Y) = H(X,Y) - H(Y)$ are nonsymmetric, however the absence of symmetry is not due to information flow but because of the different individual entropies.

Since mutual information measures the deviation from independence of the variables, it has been proposed as a tool to measure the relationship between financial market segments. Further, mutual information is nonparametric and depends on higher moments of the probability distributions of the variables, unlike correlation, which depends on the first two moments only. However, mutual information is a symmetric measure and does not contain dynamical information or directional sense. Some authors, for example Vastano and Swinney [1988], have proposed the introduction of time delay in one of the variables while computing mutual information and the use of such time delayed mutual information to define velocity of information transport in spatio-temporal systems. However, time-delayed mutual information does not distinguish information actually exchanged from shared information due to a common input signal or history and therefore does not quantify the actual overlap of the information content of two variables. Further, there may be a causal relationship without detectable time delays and conversely there may be time delays that do not reflect the naively expected causal structure between the two time series. Another issue is that the estimation of time delayed mutual information calls for a large quantity of noise-free stationary data—a condition rarely met in real-world situations.

Another information theoretic measure called transfer entropy has been introduced by Thomas Schreiber [2000] to study relationships between dynamical systems. Transfer entropy is an information theoretic concept that quantifies the degree to which a dynamical process affects the transition probabilities, i.e., the dynamics of another. Transfer entropy has the properties of mutual information and also takes the dynamics of information transport into account. Transfer entropy quantifies the exchange of information between two systems, separately for both directions and conditional to common input signal. A brief explanation of transfer entropy and the computational aspects pertaining to the same are given in the Appendix. Marschinski and Kantz [2002] used an improved estimator called effective transfer entropy and concluded that the Dow Jones U.S. stock index has higher relative impact on the German stock index DAX. Back et al. [2005] applied transfer entropy on daily closing prices of 135 stocks in the New York Stock Exchange to study information flow among groups of companies and discriminate market-leading companies from market-sensitive ones.

DATA AND METHODOLOGY

In this article, the symbolic encoding method is used to compute transfer entropy between the stock and commodity derivatives markets in India. The National Stock Exchange of India (NSEIL) being the leading stock exchange of India, the 50-stock index, Nifty, is taken as the representative of the stock market. The National Commodity & Derivatives Exchange, a leading commodity derivatives exchange of India, has launched two indices, NCDEXAGRI and FUTEXAGRI. NCDEXAGRI is an equally weighted composite index of spot prices of important agricultural commodities in every subsector and is updated three times a day with price data received from various mandis[2] and spot markets. FUTEXAGRI is constructed on the basis of online prices of the nearest month expiry futures contracts traded in NCDEX, for the same basket of commodities in NCDEXAGRI. We propose to compute the transfer entropy among these three indices, the Nifty, NCDEXAGRI, and FUTEXAGRI, so that informational transfer may be analyzed between any two of the commodities spot, the commodities derivatives, and the stock markets. Due to high liquidity in both the stock and the commodity derivatives markets and the incredibly fast information transport, enabled by digital communication network, between the two markets which have a large number of closely connected participants, there is a need to look at daily data. The use of lower frequency data such as weekly or monthly observations may not adequately capture the dynamics of the fast-moving stock prices and the commodity derivatives prices.

Data on the Nifty are available on the website of NSEIL from year end 1995, and daily values of NCDEXAGRI and FUTEXAGRI are available on the website of NCDEX from June 2005, hence the data for the period from June 2005 to September 2007 are used for this study. Thus three time series, each with 575 data points, were obtained for these variables: the stock index, Nifty (X); the commodities spot index, NCDEXAGRI (Y); and the commodities derivatives index, FUTEXAGRI (Z). These price series were transformed to log returns series since such transformation satisfies additive property of the returns and makes the results invariant in spite of arbitrary scaling of the price data. Further, such transformation improves the stationary character of the time series so that meaningful analysis may be made.

EMPIRICAL RESULTS AND DISCUSSION

The symbolic encoding method partitions the range of the data set into disjoint bins and assigns a symbol to each bin, with marginal equal probability for every symbol. The transfer entropy value depends on the number of bins (S) into which the dataset is partitioned and also on the block length k chosen for the transferee variable and the block length l for the transferor variable (however, l is chosen to be 1 generally). Hence transfer entropy $T_{Z \to Y}$ from commodity derivatives (Z) to commodity spot (Y) is computed for the number of bins S ranging from 2 to 8, the block length k of Y ranging from 1 to 10 and the block length l of Z equal to 1. Similarly, transfer entropy $T_{Y \to Z}$ from commodity spot (Y) to commodity derivatives (Z) is computed for the number of bins ranging from 2 to 8, the block length for Z ranging from 1 to 10, and the block length for Y equal to 1. Such transfer entropy values between the commodity spot and derivatives markets for the period from June 2005 to September 2007 are presented in Exhibit 1. The transfer entropy values between commodity spot and stock markets are given in Exhibit 2. The transfer entropy values between commodity derivatives and stock markets are given in Exhibit 3.

The transfer entropy in all cases behaves reasonably for partitions S = 4, 5, 6, 7, 8 of the data analyzed and for block length of the transferee series $k \geq 3$. Further, in order to consider appropriate values of k, the mutual information of the three time series containing the values of Nifty (X), NCDEXAGRI (Y), and FUTEXAGRI (Z), for delays ranging from 1 day to 20 days are computed and given in Exhibit 4. It may be observed that the first minimum has occurred for k = 2, 3, and 4 respectively. Hence, meaningful results may be obtained from transfer entropy values computed for partitions S = 4, 5, 6, 7, 8 and block length of the transferee series $k \geq 4$. For interpreting the transfer entropy values, three measures—net information flow (NIF), normalized directionality index (d), and relative explanation added (REA)—which are defined in the Appendix, have been computed and given in the respective tables.

From the transfer entropy values, a flow of information from day t of one market to day t + 1 of the other two

Exhibit 1

Commodity Derivatives and Spot Markets

		FUTEXAGRI (Z) to NCDEXAGRI (Y)				NCDEXAGRI (Y) to FUTEXAGRI (Z)					
Bins	Block	h(Y,Z)	h(Y)	T(Z->Y)	REA(Z->Y)	h(Z,Y)	h(Z)	T(Y->Z)	REA(Y->Z)	NIF(Z->Y)	d(Y,Z)
2	1	0.964796	0.991233	0.026437	0.0266708	0.995021	0.996298	0.001276	0.0012807	0.025161	0.907913254
2	2	0.957691	0.988932	0.031241	0.0315906	0.993717	0.996026	0.00231	0.0023192	0.028931	0.862299186
2	3	0.953114	0.986098	0.032984	0.033449	0.986257	0.995399	0.009141	0.0091833	0.023843	0.566005935
2	4	0.93705	0.979216	0.042166	0.043061	0.976222	0.989756	0.013534	0.0136741	0.028632	0.514039497
2	5	0.896833	0.955616	0.058784	0.0615142	0.948712	0.986449	0.037737	0.0382554	0.021047	0.218056174
2	6	0.807908	0.911569	0.103662	0.1137182	0.856285	0.955284	0.098999	0.1036331	0.004663	0.023008867
2	7	0.617054	0.803198	0.186143	0.2317523	0.687603	0.869514	0.181911	0.20921	0.004232	0.011498313
2	8	0.394818	0.581538	0.18672	0.3210796	0.43709	0.644433	0.207343	0.3217449	−0.020623	−0.05233427
2	9	0.238947	0.398074	0.159127	0.3997423	0.228355	0.361839	0.133484	0.3689044	0.025643	0.08763512
2	10	0.148312	0.270216	0.121904	0.4511354	0.117491	0.179582	0.062091	0.3457529	0.059813	0.325079486
3	1	1.507158	1.552344	0.045185	0.0291076	1.568974	1.581231	0.012256	0.0077509	0.032929	0.573266482
3	2	1.457165	1.523794	0.066629	0.0437257	1.527417	1.571639	0.044222	0.0281375	0.022407	0.202136201
3	3	1.279649	1.457905	0.178256	0.1222686	1.369934	1.511138	0.141205	0.0934428	0.037051	0.115979728
3	4	0.874069	1.272292	0.398222	0.3129958	0.960127	1.360366	0.400239	0.2942142	−0.002017	−0.00252611
3	5	0.466171	0.929889	0.463718	0.498681	0.500091	0.866797	0.366707	0.4230598	0.097011	0.116820905
3	6	0.162721	0.47675	0.31403	0.658689	0.218588	0.424867	0.206279	0.4855143	0.107751	0.207090402
3	7	0.043891	0.221264	0.177373	0.8016352	0.077144	0.148067	0.070924	0.4789994	0.106449	0.428716416
3	8	0.014185	0.08999	0.075805	0.8423714	0.03546	0.056738	0.021277	0.3750044	0.054528	0.561669516
3	9	0.003546	0.028368	0.024822	0.875	0.014185	0.021276	0.007092	0.3333333	0.01773	0.555555556
3	10	0	0.014185	0.014185	1	0.003546	0.003546	0	0	0.014185	1
4	1	1.892944	1.96928	0.076336	0.0387634	1.943596	1.988591	0.044995	0.0226266	0.031341	0.258309913
4	2	1.675602	1.918674	0.243072	0.1266875	1.759322	1.947945	0.188623	0.0968318	0.054449	0.126128401
4	3	1.018655	1.685279	0.666625	0.3955576	1.103738	1.723981	0.620244	0.3597743	0.046381	0.036041742
4	4	0.420382	1.010551	0.590169	0.5840071	0.387306	0.998184	0.610878	0.6119894	−0.020709	−0.01724246
4	5	0.126791	0.398413	0.271622	0.6817599	0.10685	0.383113	0.276263	0.7211006	−0.004641	−0.00847076
4	6	0.042555	0.114815	0.07226	0.6293603	0.024823	0.086445	0.061622	0.7128463	0.010638	0.07945803
4	7	0.010638	0.024822	0.014184	0.5714286	0	0.017731	0.017731	1	−0.003547	−0.11113896
4	8	0.003547	0.007092	0.003546	0.5	0	0	0	NA	0.003546	1
4	9	0.003547	0.007092	0.003546	0.5	0	0	0	NA	0.003546	1
4	10	0	0	0	NA	0	0	0	NA	0	NA
5	1	2.142648	2.28875	0.146102	0.0638348	2.19407	2.300939	0.106869	0.0464458	0.039233	0.155088923
5	2	1.543819	2.149866	0.606047	0.2818999	1.622654	2.185847	0.563193	0.2576544	0.042854	0.036651158
5	3	0.661267	1.555506	0.894238	0.5748856	0.691087	1.603141	0.912054	0.5689169	−0.017816	−0.0098633
5	4	0.140974	0.658754	0.51778	0.785999	0.146727	0.546949	0.400223	0.7317373	0.117557	0.128057316

5	5	0.028369	0.139636	0.111267	0.7968361	0.024822	0.156496	0.131674	0.8413889	–0.020407	–0.08399982
5	6	0	0.024822	0.024822	1	0	0.024823	0.024823	1	–0.000001	–0.00000001
5	7	0	0.003547	0.003547	1	0	0.007092	0.007092	1	–0.003545	–0.33320801
5	8	0	0	0	NA	0	0	0	NA	0	NA
5	9	0	0	0	NA	0	0	0	NA	0	NA
5	10	0	0	0	NA	0	0	0	NA	0	NA
6	1	2.257476	2.526744	0.269269	0.1065676	2.334964	2.551707	0.216743	0.0849404	0.052526	0.108075521
6	2	1.365308	2.270288	0.904981	0.3986195	1.419446	2.37573	0.956285	0.4025226	–0.051304	–0.02756403
6	3	0.401962	1.346492	0.94453	0.7014746	0.407536	1.280855	0.873319	0.681825	0.071211	0.039173221
6	4	0.089991	0.33474	0.244749	0.7311615	0.073599	0.284623	0.211024	0.7414158	0.033725	0.073995169
6	5	0.017731	0.070922	0.053191	0.749993	0.014184	0.053191	0.039007	0.7333383	0.014184	0.153842817
6	6	0.003547	0.007092	0.003546	0.5	0.003547	0.010638	0.007092	0.6666667	–0.003546	–0.33333333
6	7	0	0	0	NA	0	0.003547	0.003547	1	–0.003547	–1
6	8	0	0	0	NA	0	0	0	NA	0	NA
6	9	0	0	0	NA	0	0	0	NA	0	NA
6	10	0	0	0	NA	0	0	0	NA	0	NA
7	1	2.356776	2.741844	0.385068	0.1404413	2.363341	2.767436	0.404095	0.1460178	–0.019027	–0.02411035
7	2	1.116447	2.351417	1.234969	0.525202	1.14999	2.318683	1.168694	0.5040335	0.066275	0.027572501
7	3	0.239395	0.998057	0.758662	0.7601389	0.279518	1.009968	0.73045	0.7232407	0.028212	0.018945519
7	4	0.039007	0.208819	0.169812	0.8132019	0.03546	0.193296	0.157836	0.8165508	0.011976	0.036551421
7	5	0.010639	0.031915	0.021276	0.6666458	0.007092	0.042554	0.035461	0.8333177	–0.014185	–0.25001322
7	6	0	0.003547	0.003547	1	0	0	0	NA	0.003547	1
7	7	0	0	0	NA	0	0	0	NA	0	NA
7	8	0	0	0	NA	0	0	0	NA	0	NA
7	9	0	0	0	NA	0	0	0	NA	0	NA
7	10	0	0	0	NA	0	0	0	NA	0	NA
8	1	2.327146	2.940391	0.613245	0.208559	2.334021	2.930249	0.596229	0.2034738	0.017016	0.014068926
8	2	0.890742	2.328055	1.437313	0.6173879	0.898853	2.324353	1.4255	0.6132889	0.011813	0.004126361
8	3	0.14539	0.739737	0.594347	0.8034572	0.16272	0.743926	0.581206	0.7812686	0.013141	0.011178569
8	4	0.024822	0.104175	0.079352	0.7617183	0.031915	0.116153	0.084238	0.7252331	–0.004886	–0.02986735
8	5	0.003547	0.024823	0.021276	0.8571083	0.007092	0.01773	0.010637	0.5999436	0.010639	0.333375114
8	6	0	0	0	NA	0	0.003547	0.003547	1	–0.003547	–1
8	7	0	0	0	NA	0	0	0	NA	0	NA
8	8	0	0	0	NA	0	0	0	NA	0	NA
8	9	0	0	0	NA	0	0	0	NA	0	NA
8	10	0	0	0	NA	0	0	0	NA	0	NA

Exhibit 2
Commodity Spot and Stock Markets

		NCDEXAGRI (Y) to NIFTY (X)				NIFTY (X) to NCDEXAGRI (Y)					
Bins	Block	h(X,Y)	h(X)	T(Y->X)	REA(Y->X)	h(Y,X)	h(Y)	T(X->Y)	REA(X->Y)	NIF(Y->X)	d(X,Y)
2	1	0.999585	0.999846	0.000261	0.000261	0.986564	0.991233	0.004669	0.00471	–0.00441	–0.89412
2	2	0.998124	0.999359	0.001235	0.001236	0.983084	0.988932	0.005848	0.005913	–0.00461	–0.65128
2	3	0.990292	0.998531	0.008238	0.00825	0.977217	0.986098	0.008881	0.009006	–0.00064	–0.03756
2	4	0.966664	0.989682	0.023018	0.023258	0.956574	0.979216	0.022641	0.023122	0.000377	0.008257
2	5	0.923974	0.970726	0.046753	0.048163	0.915256	0.955616	0.040361	0.042236	0.006392	0.073375
2	6	0.830558	0.938004	0.107447	0.114549	0.813894	0.911569	0.097675	0.10715	0.009772	0.04764
2	7	0.640989	0.832155	0.191166	0.229724	0.597772	0.803198	0.205426	0.25576	–0.01426	–0.03596
2	8	0.354747	0.61525	0.260503	0.42341	0.368768	0.581538	0.21277	0.365875	0.047733	0.100857
2	9	0.178712	0.377139	0.198427	0.526138	0.247894	0.398074	0.15018	0.377267	0.048247	0.138399
2	10	0.094875	0.192894	0.098019	0.50815	0.154757	0.270216	0.115458	0.42728	–0.01744	–0.08169
3	1	1.563854	1.579512	0.015658	0.009913	1.53408	1.552344	0.018263	0.011765	–0.00261	–0.0768
3	2	1.500995	1.547676	0.046682	0.030163	1.473953	1.523794	0.049841	0.032708	–0.00316	–0.03273
3	3	1.323054	1.493361	0.170306	0.114042	1.293003	1.457905	0.164902	0.113109	0.005404	0.016121
3	4	0.899231	1.318121	0.41889	0.317793	0.882579	1.272292	0.389713	0.306308	0.029177	0.036083
3	5	0.447949	0.887578	0.439629	0.495313	0.43568	0.929889	0.494209	0.531471	–0.05458	–0.05845
3	6	0.175567	0.444979	0.269412	0.605449	0.143649	0.47675	0.333101	0.698691	–0.06369	–0.10571
3	7	0.070921	0.181789	0.110868	0.609872	0.052321	0.221264	0.168942	0.763531	–0.05807	–0.20755
3	8	0.021276	0.070922	0.049645	0.699994	0.014185	0.08999	0.075805	0.842371	–0.02616	–0.20853
3	9	0.007092	0.021277	0.014185	0.666682	0.003546	0.028368	0.024822	0.875	–0.01064	–0.27269
3	10	0.003547	0.007092	0.003546	0.5	0	0.014185	0.014185	1	–0.01064	–0.60002
4	1	1.926498	1.977431	0.050932	0.025757	1.915572	1.96928	0.053708	0.027273	–0.00278	–0.02653
4	2	1.662118	1.912159	0.250041	0.130764	1.682228	1.918674	0.236445	0.123234	0.013596	0.027947
4	3	0.988593	1.636838	0.648245	0.396035	1.026938	1.685279	0.658341	0.390642	–0.0101	–0.00773
4	4	0.370916	1.014262	0.643346	0.6343	0.406598	1.010551	0.603953	0.597647	0.039393	0.031583
4	5	0.106853	0.434276	0.327423	0.753951	0.112605	0.398413	0.285808	0.717366	0.041615	0.067862
4	6	0.028369	0.140974	0.112605	0.798764	0.046101	0.114815	0.068714	0.598476	0.043891	0.242065
4	7	0.010638	0.024822	0.014184	0.571429	0.007092	0.024822	0.01773	0.714286	–0.00355	–0.11111
4	8	0.003547	0.010639	0.007092	0.666604	0	0.007092	0.007092	1	0	0
4	9	0	0	0	NA	0	0.007092	0.007092	1	–0.00709	–1
4	10	0	0	0	NA	0	0	0	NA	0	NA
5	1	2.192635	2.272418	0.079784	0.03511	2.175169	2.28875	0.11358	0.049625	–0.0338	–0.17478
5	2	1.580817	2.16092	0.580103	0.268452	1.521086	2.149866	0.62878	0.292474	–0.04868	–0.04027
5	3	0.608234	1.569467	0.961232	0.612458	0.591488	1.555506	0.964018	0.619746	–0.00279	–0.00145
5	4	0.156965	0.608822	0.451857	0.742182	0.157367	0.658754	0.501388	0.761116	–0.04953	–0.05196

5	5	0.024822	0.147842	0.12302	0.832105	0.031915	0.139636	0.107721	0.771441	0.015299	0.066304
5	6	0.003546	0.052322	0.048777	0.932246	0.003546	0.024822	0.021276	0.857143	0.027501	0.392574
5	7	0.003547	0.014185	0.010638	0.749947	0	0.003547	0.003547	1	0.007091	0.499894
5	8	0	0	0	NA	0	0	0	NA	0	NA
5	9	0	0	0	NA	0	0	0	NA	0	NA
5	10	0	0	0	NA	0	0	0	NA	0	NA
6	1	2.287804	2.500175	0.212371	0.084942	2.270616	2.526744	0.256128	0.101367	–0.04376	–0.0934
6	2	1.296995	2.288172	0.991177	0.433174	1.241806	2.270288	1.028483	0.453019	–0.03731	–0.01847
6	3	0.336032	1.267249	0.931217	0.734833	0.324971	1.346492	1.021521	0.758654	–0.0903	–0.04624
6	4	0.056738	0.36795	0.311213	0.845802	0.063829	0.33474	0.27091	0.809315	0.040303	0.069235
6	5	0.007092	0.106852	0.099759	0.933618	0.014184	0.070922	0.056738	0.800006	0.043021	0.2749
6	6	0.003546	0.014184	0.010638	0.75	0	0.007092	0.007092	1	0.003546	0.2
6	7	0	0.003547	0.003547	1	0	0	0	NA	0.003547	1
6	8	0	0	0	NA	0	0	0	NA	0	NA
6	9	0	0	0	NA	0	0	0	NA	0	NA
6	10	0	0	0	NA	0	0	0	NA	0	NA
7	1	2.343438	2.711665	0.368227	0.135794	2.335298	2.741844	0.406546	0.148275	–0.03832	–0.04946
7	2	0.99162	2.286609	1.294989	0.566336	0.983317	2.351417	1.368099	0.581819	–0.07311	–0.02745
7	3	0.237189	1.012999	0.77581	0.765855	0.209287	0.998057	0.788771	0.790307	–0.01296	–0.00828
7	4	0.039007	0.250973	0.211966	0.844577	0.028368	0.208819	0.180451	0.86415	0.031515	0.08031
7	5	0.003546	0.059414	0.055868	0.940317	0	0.031915	0.031915	1	0.023953	0.272866
7	6	0	0.014185	0.014185	1	0	0.003547	0.003547	1	0.010638	0.599932
7	7	0	0	0	NA	0	0	0	NA	0	NA
7	8	0	0	0	NA	0	0	0	NA	0	NA
7	9	0	0	0	NA	0	0	0	NA	0	NA
7	10	0	0	0	NA	0	0	0	NA	0	NA
8	1	2.292738	2.881165	0.588427	0.204232	2.312346	2.940391	0.628046	0.213593	–0.03962	–0.03257
8	2	0.780281	2.240476	1.460195	0.651734	0.721894	2.328055	1.606161	0.689915	–0.14597	–0.0476
8	3	0.160042	0.825438	0.665397	0.806114	0.125451	0.739737	0.614285	0.83041	0.051112	0.039941
8	4	0.039006	0.158304	0.119298	0.753601	0.024822	0.104175	0.079352	0.761718	0.039946	0.201087
8	5	0.007092	0.031915	0.024822	0.777753	0.003547	0.024823	0.021276	0.857108	0.003546	0.076923
8	6	0	0	0	NA	0	0	0	NA	0	NA
8	7	0	0	0	NA	0	0	0	NA	0	NA
8	8	0	0	0	NA	0	0	0	NA	0	NA
8	9	0	0	0	NA	0	0	0	NA	0	NA
8	10	0	0	0	NA	0	0	0	NA	0	NA

EXHIBIT 3
Commodity Derivatives and Stock Markets

		FUTEXAGRI (Z) to NIFTY (X)				NIFTY (X) to FUTEXAGRI (Z)					
Bins	Block	h(X,Z)	h(X)	T(Z–>X)	REA(Z–>X)	h(Z,X)	h(Z)	T(X–>Z)	REA(X–>Y)	NIF(X–>Z)	d(X,Z)
2	1	0.997228	0.999846	0.002618	0.002618	0.991442	0.996298	0.004856	0.004874	–0.00224	–0.29944
2	2	0.992949	0.999359	0.006411	0.006415	0.989496	0.996026	0.00653	0.006556	–0.00012	–0.0092
2	3	0.981955	0.998531	0.016576	0.0166	0.98562	0.995399	0.009779	0.009824	0.006797	0.257902
2	4	0.957798	0.989682	0.031884	0.032216	0.971564	0.989756	0.018192	0.01838	0.013692	0.273424
2	5	0.916538	0.970726	0.054189	0.055823	0.948591	0.986449	0.037858	0.038378	0.016331	0.17742
2	6	0.815618	0.938004	0.122387	0.130476	0.855233	0.955284	0.100051	0.104734	0.022336	0.100414
2	7	0.619467	0.832155	0.212688	0.255587	0.675426	0.869514	0.194089	0.223215	0.018599	0.045723
2	8	0.376094	0.61525	0.239156	0.388714	0.410683	0.644433	0.23375	0.362722	0.005406	0.011431
2	9	0.204877	0.377139	0.172262	0.45676	0.199518	0.361839	0.162321	0.4486	0.009941	0.029712
2	10	0.109529	0.192894	0.083364	0.432175	0.075808	0.179582	0.103774	0.577864	–0.02041	–0.10906
3	1	1.568259	1.579512	0.011253	0.007124	1.565479	1.581231	0.015751	0.009961	–0.0045	–0.16657
3	2	1.5223	1.547676	0.025377	0.016397	1.536649	1.571639	0.03499	0.022263	–0.00961	–0.15924
3	3	1.357121	1.493361	0.136239	0.09123	1.364861	1.511138	0.146278	0.0968	–0.01004	–0.03553
3	4	0.891093	1.318121	0.427029	0.323968	0.912951	1.360366	0.447415	0.328893	–0.02039	–0.02331
3	5	0.468936	0.887578	0.418642	0.471668	0.425045	0.866797	0.441752	0.509637	–0.02311	–0.02686
3	6	0.204405	0.444979	0.240574	0.540641	0.170681	0.424867	0.254186	0.598272	–0.01361	–0.02751
3	7	0.088653	0.181789	0.093137	0.512336	0.067376	0.148067	0.080691	0.544963	0.012446	0.0716
3	8	0.042553	0.070922	0.028369	0.400003	0.017731	0.056738	0.039007	0.687493	–0.01064	–0.15789
3	9	0.003547	0.021277	0.017731	0.833341	0.007092	0.021276	0.014184	0.666667	0.003547	0.111139
3	10	0	0.007092	0.007092	1	0	0.003546	0.003546	1	0.003546	0.333333
4	1	1.939575	1.977431	0.037855	0.019144	1.942383	1.988591	0.046208	0.023237	–0.00835	–0.09937
4	2	1.701593	1.912159	0.210566	0.11012	1.718065	1.947945	0.229879	0.118011	–0.01931	–0.04385
4	3	1.011594	1.636838	0.625244	0.381983	0.993785	1.723981	0.730196	0.423552	–0.10495	–0.07743
4	4	0.405329	1.014262	0.608933	0.600371	0.322364	0.998184	0.67582	0.67705	–0.06689	–0.05206
4	5	0.104175	0.434276	0.330101	0.760118	0.08069	0.383113	0.302423	0.789383	0.027678	0.043758
4	6	0.0461	0.140974	0.094874	0.672989	0.024823	0.086445	0.061622	0.712846	0.033252	0.212478
4	7	0.003547	0.024822	0.021276	0.857143	0.003547	0.017731	0.014184	0.799955	0.007092	0.2
4	8	0.003547	0.010639	0.007092	0.666604	0	0	0	NA	0.007092	1
4	9	0	0	0	NA	0	0	0	NA	0	NA
4	10	0	0	0	NA	0	0	0	NA	0	NA
5	1	2.17804	2.272418	0.094378	0.041532	2.185691	2.300939	0.115247	0.050087	–0.02087	–0.09955
5	2	1.573561	2.16092	0.587359	0.27181	1.569372	2.185847	0.616475	0.28203	–0.02912	–0.02419
5	3	0.600539	1.569467	0.968927	0.617361	0.554512	1.603141	1.048629	0.654109	–0.0797	–0.0395
5	4	0.13522	0.608822	0.473602	0.777899	0.104176	0.546949	0.442774	0.809534	0.030828	0.033641

5	5	0.031915	0.147842	0.115928	0.784134	0.031915	0.156496	0.124581	0.796065	–0.00865	–0.03598
5	6	0.010638	0.052322	0.041684	0.796682	–0.000001	0.024823	0.024824	1	0.01686	0.253503
5	7	0	0.014185	0.014185	1	0	0.007092	0.007092	1	0.007093	0.333365
5	8	0	0	0	NA	0	0	0	NA	0	NA
5	9	0	0	0	NA	0	0	0	NA	0	NA
5	10	0	0	0	NA	0	0	0	NA	0	NA
6	1	2.266243	2.500175	0.233932	0.093566	2.332613	2.551707	0.219094	0.085862	0.014838	0.032753
6	2	1.293224	2.288172	0.994948	0.434822	1.314047	2.37573	1.061683	0.446887	–0.06673	–0.03245
6	3	0.343639	1.267249	0.923609	0.72883	0.318525	1.280855	0.962329	0.751318	–0.03872	–0.02053
6	4	0.068715	0.36795	0.299235	0.813249	0.061623	0.284623	0.223001	0.783496	0.076234	0.145976
6	5	0.014185	0.106852	0.092667	0.867246	0.003547	0.053191	0.049644	0.933316	0.043023	0.302317
6	6	0.003547	0.014184	0.010637	0.749929	0.003547	0.010638	0.007092	0.666667	0.003545	0.199955
6	7	0	0.003547	0.003547	1	0	0.003547	0.003547	1	0	0
6	8	0	0	0	NA	0	0	0	NA	0	NA
6	9	0	0	0	NA	0	0	0	NA	0	NA
6	10	0	0	0	NA	0	0	0	NA	0	NA
7	1	2.335739	2.711665	0.375926	0.138633	2.361566	2.767436	0.405869	0.146659	–0.02994	–0.0383
7	2	0.983408	2.286609	1.303201	0.569927	0.922678	2.318683	1.396005	0.602068	–0.0928	–0.03438
7	3	0.210156	1.012999	0.802843	0.792541	0.176905	1.009968	0.833063	0.824841	–0.03022	–0.01847
7	4	0.03546	0.250973	0.215512	0.858706	0.039007	0.193296	0.154289	0.798201	0.061223	0.165557
7	5	0.007092	0.059414	0.052321	0.880617	0.010638	0.042554	0.031916	0.750012	0.020405	0.242233
7	6	0	0.014185	0.014185	1	0	0	0	NA	0.014185	1
7	7	0	0	0	NA	0	0	0	NA	0	NA
7	8	0	0	0	NA	0	0	0	NA	0	NA
7	9	0	0	0	NA	0	0	0	NA	0	NA
7	10	0	0	0	NA	0	0	0	NA	0	NA
8	1	2.289794	2.881165	0.591371	0.205254	2.327457	2.930249	0.602792	0.205714	–0.01142	–0.00956
8	2	0.716939	2.240476	1.523537	0.680006	0.726304	2.324353	1.598049	0.687524	–0.07451	–0.02387
8	3	0.121036	0.825438	0.704403	0.853369	0.130335	0.743926	0.613591	0.824801	0.090812	0.068902
8	4	0.010638	0.158304	0.147666	0.9328	0.014185	0.116153	0.101968	0.877877	0.045698	0.18306
8	5	0.003547	0.031915	0.028368	0.888861	0.003546	0.01773	0.014184	0.8	0.014184	0.333333
8	6	0	0	0	NA	0	0.003547	0.003547	1	–0.00355	–1
8	7	0	0	0	NA	0	0	0	NA	0	NA
8	8	0	0	0	NA	0	0	0	NA	0	NA
8	9	0	0	0	NA	0	0	0	NA	0	NA
8	10	0	0	0	NA	0	0	0	NA	0	NA

EXHIBIT 4
Mutual Information (MI)

Delay	NIFTY-MI	NCDEXAGRI-MI	FUTEXAGRI-MI
1	1.476563	1.390625	1.296875
2	1.410156	1.375	1.480469
3	1.476563	1.300781	1.546875
4	1.382813	1.464844	1.265625
5	1.363281	1.476563	1.367188
6	1.40625	1.292969	1.441406
7	1.410156	1.304688	1.429688
8	1.535156	1.378906	1.261719
9	1.429688	1.46875	1.382813
10	1.359375	1.214844	1.355469
11	1.429688	1.371094	1.421875
12	1.386719	1.359375	1.441406
13	1.410156	1.402344	1.460938
14	1.367188	1.402344	1.3125
15	1.464844	1.457031	1.425781
16	1.375	1.433594	1.410156
17	1.324219	1.460938	1.4375
18	1.34375	1.34375	1.40625
19	1.472656	1.492188	1.417969
20	1.480469	1.421875	1.359375

markets is observed, which suggests interactions among the three markets at a time scale of a day or less. The information flows between any two markets in both the directions are more or less at the same level, when up to six past values of the transferee series are considered, and hence in such cases, the NIF values are not significant. Also, the REA in such cases either increases or remains at high levels, thereby implying that whatever information flows from one market towards the prediction of the next price in the other market cannot be compensated by the inclusion of more numbers of past values realized by the transferee market, up to six days. Further, the absolute value of d has been generally less than 0.33 except in a few cases, indicating that feedbacks in both directions between any two markets do not vary much.

If the time series are partitioned into four or more bins and when seven or more past values of the transferee market are considered (i.e., $k \geq 7$), even the entropy rates (given lagged values of the same market only) and the conditional entropy rates (given lagged values of both the same and the transferor markets) approach or become zero in respect of all the markets and hence the transfer entropy between any two markets approaches or becomes zero. Hence, price data beyond six days in any market do not have significant informational value in the same market nor in any other market.

Thus the results obtained across the markets are more or less consistent and reiterate that

- There exist interactions between any two markets, with up to six-days-old price information, and the feedback between any two markets is almost at the same level in both directions.
- Information generation in the markets tend to zero if seven or more past values are considered.

CONCLUSION

Entropic analysis is a novel area in the Indian financial market, and there is a lot of scope for the application of entropic analysis in the Indian markets. This article applies entropic analysis to study interaction between commodities and stock markets, and transfer entropy is found to be suited for this study. Transfer entropy values among commodities spot, commodities derivatives and stock markets in India for the period June 2005 through September 2007 were computed, and it was found that interactions existed between any two markets. It may be noted that transfer entropy quantifies information transmission, including nonlinear dynamic relationship, and thus transfer entropy proves to be a promising measure to identify directional information. It may further be noted that in the computation of transfer entropy, determination of the appropriate partition of the data series and the block length of the transferee time series has to be done with utmost care.

APPENDIX

BASIC CONCEPTS OF ENTROPY

1. The *entropy* of a random variable X with p(x) as the probability mass function is defined (according to the Shannon approach) as $H(X) = H(p) = -\Sigma_x p(x) \log = E[\log \{1/p(x)\}]$ where the base of the logarithm is 2 and log 0 is taken as 0. Entropy is measured in bits and $0 \leq H(X) < \infty$. If the logarithm is taken to the base e, then entropy is measured in "nats." Entropy of a dynamical system is the amount of disorder in the system, as described in thermodynamics, and also is the amount of information needed to predict

the next measurement with a certain precision, as described in information theory. Entropy does not measure the shape of the distribution of the realizations of a system but provides information about how the system fluctuates with time—in frequency space or phase space.

2. The *joint entropy* of a pair of random variables X and Y with a joint probability mass function p(x,y) is defined as $H(X,Y) = -\Sigma_x \Sigma_y p(x,y) \log p(x,y) = -E[\log p(x,y)]$.
3. The *conditional entropy* of a random variable Y given another variable X is defined as $H(Y/X) = \Sigma_x p(x,y) \log p(Y/X = x) = -E[\log p(Y/X)$. Then we get the chain rule $H(X,Y) = H(X) + H(Y/X) = H(Y) + H(X/Y)$. Conditioning reduces entropy, i.e., $H(X/Y) \leq H(X)$ with equality if X and Y are independent. It follows that $H(X,Y) \leq H(X) + H(Y)$ with equality if X and Y are independent.
4. The *relative entropy or cross entropy or Kullback–Leibler (KL) distance* between two probability functions p(x) and q(x) is $D(p \,||\, q) = \Sigma_x p(x) \log \{p(x)/q(x)\} = E[\log \{[\log \{p(x)/q(x)\}]$.

   It may be noted that $D(p \,||\, q) \geq 0 = 0$ if $p = q$.

   However, $D(p \,||\, q) \neq D(q \,||\, p)$ in general.

   ∵ relative entropy is not symmetric and does not satisfy the triangle property, it is not a true distance between distributions.
5. The *mutual information* I(X;Y) between two random variables X and Y with a joint probability mass function p(x,y) and marginal mass functions p(x) and p(y), is defined as the relative entropy between the joint distribution p(x,y) and the product distribution p(x) p(y).

   That is, $I(X;Y) = D(p(x,y)||p(x)\, p(y)) = \Sigma_x \Sigma_y p(x,y)/p(x)\, p(y)\}$

   It may be noted that $I(X;Y) \geq 0 = 0$ if X and Y are independent.

   Also, $I(X;Y) = H(X) - H(X/Y) = H(Y) - H(Y/X)$ where H denotes the entropy.

That is, mutual information is the reduction in the uncertainty of X due to the knowledge of Y and vice versa. Due to symmetry, X says as much about Y as Y says about X.

Also, $I(X;Y) = H(X) + H(Y) - H(X,Y)$ and $I(X;X) = H(X)$. Thus the mutual information of a random variable with itself is the entropy of the random variable. That is why entropy is referred to as self-information.

TRANSFER ENTROPY

The rate at which the entropy of a stochastic process X_n, n = 1, 2,... grows with n is given by

$$\begin{aligned}
h_n(X) &= -\sum p(x_{n+1}) \log p\,(x_{n+1}/\, x_n, x_{n-1}, \ldots, x_1) \\
&= -\sum p(x_{n+1}) \log \{p(x_{n+1}, x_n, x_{n-1}, \ldots, x_1) \\
&\qquad \times p(x_n, x_{n-1}, \ldots, x_1)\} \\
&= -\sum p(x_{n+1}) \log p(x_{n+1}, x_n, x_{n-1}, \ldots, x_1) \\
&\quad + \sum p(x_{n-1}) \log p(x_n, x_{n-1}, \ldots, x_1) \\
&= H_{n+1}(X) - H_n(X)
\end{aligned}$$

where $H_n(X)$ is the entropy of the process given by n dimensional delay vectors constructed from X_n. Thus, $h_n(X)$ denotes the information still transmitted by x_{n+1} when $x_1, x_2, \ldots, x_n$ are known or the missing information required to forecast x_{n+1} using $x_1, x_2, \ldots, x_n$. Alternatively, $-h_n(X)$ denotes the information known about x_{n+1} from $x_1, x_2, \ldots, x_n$.

The generalization of the entropy rate to construct mutual information rate between two variables (X, Y) is done using the generalized Markov property,

$$p(x_{n+1}/x_n, x_{n-1}, \ldots, x_{n-k+1}) = p(x_{n+1}/x_n, x_{n-1}, \ldots, x_{n-k+1}, \times y_n, y_{n-1}, \ldots, y_{n-l+1})$$

where k and l denote the number of past observations included in the variables X and Y respectively. In the absence of information flow from Y to X, the state of Y has no influence on the transition probabilities of X. Just as mutual information is quantified as the deviation from the independence of the variables X and Y and is defined as the relative entropy between the joint distribution p(x,y) and the product distribution p(x) p(y), the mutual information rate is quantified as the deviation from the independence of the entropy rates and is defined as the relative entropy between the transition probabilities $p\,(x_{n+1}/x_n, x_{n-1}, \ldots, x_{n-k+1}, y_n, y_{n-1}, \ldots, y_{n-l+1})$ and $p\,(x_{n+1}/ x_n, x_{n-1}, \ldots, x_{n-k+1})$. This is termed as transfer entropy and denoted as $T_{Y\to X}$. If k and l denote block lengths taken in the variables X and Y respectively, then

$$\begin{aligned}
&T_{Y\to X}(k, l) \\
&\quad = \Sigma p(x_{n+1}, x_n, x_{n-1}, \ldots, x_{n-k+1}, y_n, y_{n-1}, \ldots, y_{n-l+1}) \\
&\quad \times \log\{p(x_{n+1}/x_n, x_{n-1}, \ldots, x_{n-k+1}, y_n, \\
&\qquad \times y_{n-1}, \ldots, y_{n-l+1})/p(x_{n+1}/x_n, x_{n-1}, \ldots, x_{n-k+1})\} \\
&\quad = -H_{k+1,l}(X,Y) + H_{k,l}(X,Y) + H_{k+1}(X) - H_k(X) \\
&\quad = h_k(X) - h_{k,l}(X,Y).
\end{aligned}$$

Obviously, $0 \le T_{Y \to X}(k, l) \le H(X)$. Also $T_{Y \to X}$ is asymmetric and takes into account only statistical dependencies originating in the variable Y and not those deriving from a common input signal. Further, transfer entropy is closely related to conditional entropy extended to two variables X and Y and may be explained as follows.

Transfer entropy = (Information about future observation x_{n+1} gained from past observations of X_n and Y_n) − (Information about future observation x_{n+1} gained from past observations of X_n only) = Information flow from Y_n to X_n.

COMPUTATIONAL ASPECTS

The computation of transfer entropy from a time series to another may be done in two ways:

1. The symbolic encoding method divides the range of the dataset into S disjoint intervals such that the number of data points in every interval is constant and assigns one symbol to each interval. Then $p(x_n) = 1/S$ so that $H(X) = \log_2 S$. However, determining the partition is a contentious issue, called the generating partition problem, and even for a two-dimensional deterministic system, the partition lines may exhibit considerably complicated geometry.
2. The correlation integral method computes the fraction of data points lying within boxes of constant size $\in$, after embedding the dataset into an appropriate phase space, and uses the formula $H_n(X, 2\in) \sim -\log_2\{C_n(X, \in)\}$ where C_n is the generalized correlation integral of order n. However, determining the box size $\in$ remains as a contentious issue. The parameter $\in$ plays the role of defining the resolution or the scale of concerns, just as the number of symbols S does in the symbolic encoding method.

The symbolic encoding method has the advantage of neutralizing undesirable effects due to nonhomogeneous histograms, and it also ignores the trivial information gained by just observing marginal distributions. Further, for data with an approximately symmetric distribution, the concrete meaning of partitions is intuitive with S = 2 corresponding to the two possible signs of the increments and S = 3 corresponding to the three possible moves with regard to larger gain, roughly neutral, and larger loss.

For a given partition, $T_{Y \to X}(k, l)$ is a nonincreasing function of the block length k of the series X, since the inclusion of more past observations in the variable X is likely to result in reduction of flow of information from Y in the estimation of the next value of X. The parameter k is to be chosen as large as possible in order to find an invariant value for $T_{Y \to X}$, however due to the finite size of real time series, it is required to find a reasonable compromise between unwanted finite sample effects and a high value for k. Further, a very small value of k may lead to misinterpretation of information contained in past observations of both series as an information flow from Y to X and hence k may be as large as possible.

Further, in order to consider appropriate values of k, it is proposed that the concept of mutual information of a time series be used. The mutual information I(k) between a time series $\{x_1, x_2, \ldots, x_n\}$ and itself with a delay of k viz. $\{x_{k+1}, x_{k+2}, \ldots, x_n\}$ measures the information carried over by the delayed time series from the original time series. If I(k) is small or around 0, then the two time series are essentially independent and if I(k) is very large, then the delayed series is related to the original series. If the delay k is too short, then the delayed series is similar to the original series, and when the data are plotted, most of the observations will lie near the line $x_{k+i} = x_i$, and I(k) will tend to be large. If the delay k is too long, then the data are independent and no information can be gained from the plot and I(k) will tend to be small.

A good choice for k is such that contiguous templates of size k constructed from the time series are not within the neighborhood of one another. Such a choice is provided by the value of k corresponding to which the mutual information of the time series with delay k viz. I(k) is small, and consequently the contiguous templates are independent to a large extent. As k is increased, I(k) decreases and may rise again and hence the first minimum of I(k) may be considered to choose the value of k. It may also be noted that Fraser and Swinney [1986] suggested that in the construction of multidimensional phase portrait from a scalar time series, the time delay T that produces the first local minimum of the mutual information of the time series may be used. Since mutual information measures the general dependence between two variables or between two time series of the same variable with time delay, it provides a good criterion for the choice of k. Also, the choices for l are l = k or l = 1, and for computational reasons, l = 1 is preferred usually.

MEASURES FOR INTERPRETING TRANSFER ENTROPY

1. The *net information flow (NIF)* is defined to measure the disparity in influences of the two variables on each other. If $NIF_{Y \to X} = T_{Y \to X} - T_{X \to Y}$ is positive, the variable Y may be said to influence the variable X.
2. The *normalized directionality index (d)* is defined in order that relevant but small-scale causal structure is not neglected and quantified as $d(X,Y) = \frac{T_{Y \to X} - T_{X \to Y}}{T_{Y \to X} + T_{X \to Y}}$. The index varies from −1 (in case of uni-directional causality from X to Y) through 0 (in case of equal feedback between the two variables) to +1 (in case of uni-directional causality from Y to X), with intermediate values corresponding to bidirectional causality between the two variables X and Y. The index thus has the property of coefficient of correlation between two variables and also has the additional feature of directionality.
3. *The relative explanation added (REA) is defined to compare the* measured amount of information flow from Y to X with the total flow of information in X. $REA_{Y \to X}(k, l) = T_{Y \to X}(k, l)/h_k(X)$ and $REA_{X \to Y}(k, l) = T_{X \to Y}(k, l)/h_k(Y)$. The ratio $REA_{Y \to X}$ measures how much of x_{n+1} is additionally explained when the past values of X are already known and then the last value y_n of Y is taken into account. The ratio varies from 0 (in the case of no information flow at all from a variable to the other) to 1 (in the case of all the information in the current value of one variable being transferred from past values of the other variable) with intermediate values corresponding to the amount of information in one variable caused by the other variable.

ENDNOTES

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[1]The "liberalization era" refers to the economic liberalization policies started by the government in India in 1991. Prior to 1991 the industrial sector was concentrated in a few hands. Private enterprise and capital increased significantly since 1991 after the introduction of the liberalization policies.

[2]Electronic portals that enhance price discovery.

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